

Annuitization: If Not Now, When?

By: Moshe A. Milevsky
Version: *July 10, 2006*

In previous writing – and related papers by Chen and Milevsky (JFP, 2003) or Ameriks, Veres and Warshawsky (JFP, 2001) – the point has been made that payout annuities, either variable or fixed, have a rightful place in the optimal life-cycle portfolio[1]. I have argued that depending on a lifecycle investor's[2] bequest motive versus their desire for consumption, they should have a substantial portion of their total wealth allocated to income annuities. Indeed, a product allocation of up to 75% of total wealth can be justified under a variety of methodological frameworks. This would imply that as much as 75% of desired (retirement) income can and should be longevity insured. Of course, for those lifecycle investors with a lucrative Defined Benefit (DB) pension and/or maximum Social Security benefits, a large portion of their retirement income might already contain longevity insurance. There is no need for them to acquire any more.

The following table (#1) illustrates the extent to which Americans' retirement income is longevity insured. A typical retiree has diverse sources of income. Many are entitled to Social Security payments, a number of them have DB pensions and some have income from other annuities. All of these sources are clearly longevity insured. They can not be outlived, although their real purchasing power might decline over time. In contrast, other sources of retirement income such as employment, interest income, dividends, systematic withdrawal plans (SWiPs), etc., are not longevity insured. Notice from the table that at advanced ages, 80% of an average retiree's income is longevity insured. And, while these numbers are population averages, they do confirm that

[1] For more-theoretical papers that make the same arguments we refer the interested reader to the classical paper by Yaari (RES, 1965) and the recent extension by Davidoff, Diamond and Brown (AER, 2005).

[2] I use the generic term "lifecycle investor" to denote an individual -- in contrast to an institutional investor or endowment -- constructing a personal portfolio to take account of employment income, health status, aging, retirement, changes in human capital, mortality, etc. The lifecycle investor is the holistic antithesis of the classical one-period mean variance optimizer, whose only concern is financial risk *vis a vis* the market portfolio.

longevity insurance is not an esoteric concept. For the average American it is the foundation of their retirement income.

Table #1

What Fraction of Elderly Income Contains Longevity Insurance which can't be Outlived?

Average for U.S. Population:

Age Group	Income in 2004
65-69	49.9%
70-74	62.4%
75-79	70.4%
80-84	75.1%
85+	80.1%

Social Security, Pensions & Annuities

47 Source: Employee Benefit Research Institute, 2006 Copyright © 2006 The IFID Centre

However, one question I have not addressed -- which is the focus of this essay -- is exactly when and how a lifecycle investor should go about purchasing these income annuities or longevity insurance policies, if they do not have them already.

Intuitively, it makes very little sense to purchase an (irreversible) *immediate income annuity* at the age of 30, 40 or even 50 for a variety of reasons. First of all, the mortality credits – the *raison d'être* of annuitization – are miniscule at this age, and real-world transaction costs, fees and expenses completely eliminate the insurance-mortality pooling effect. This is not just conjecture. The empirical money's worth ratios tell the same story. The Implied Longevity Yields (ILY) computed at the age of 50, as per the methodology developed in Milevsky (JRI, 2005) are substantially lower than the relevant risk-free rates.

For example at the age of 50 the unisex mortality rate – i.e. the probability of dying within one year – is approximately 0.4%, or 4 deaths per 1000 exposed,

according to the RP2000 (pensioners, non-projected) mortality table. Thus, during the age of 50 annuitization would only add 42 basis points of return (a.k.a mortality credits) when the underlying pricing interest rate in the economy is 5%. Note that these mortality credits are relatively insensitive to the pricing rate or the underlying interest rate in the economy. At a 7% pricing rate the mortality credits would be 43 basis points and at a 10% pricing rate they would be 44 basis points. In fact, to a crude first order of approximation, the mortality credits are slightly higher than the mortality rates themselves, as per the following Taylor series expansion:

$$\text{Mortality Credits} := \frac{1+R}{1-q_x} - (1+R) \approx q_x(1+q_x) + Rq_x(1+q_x) > q_x \qquad \text{(eq.1).}$$

And, while an additional 42 basis points of investment return is not something to be taken lightly, it is highly unlikely that any of these mortality credits will actually accrue to the annuitant (or tontine participant) once insurance company profits, commissions and transaction costs are taken into account. More importantly, any insurance company that must set aside equity capital on the order of 5% to 10% of annuity reserves will demand a return on equity on the order of 10% to 20%, which creates a drag of yet another 50 and 200 basis points, even for the most efficient, low cost providers, unless they are non-profit or perhaps government entities. Moreover, when you consider the IAM2000 table instead of the more lenient (i.e. less integrative of anti-selection) RP2000 table, the pure mortality credits at age 50 drop to about 30 basis points.

Table #2

Value of Unisex Mortality Credits:

Age of Annuitant	Spread Above Pricing Interest Rate (in Basis Points = 1/100 %)	Age of Annuitant	Spread Above Pricing Interest Rate (in Basis Points = 1/100 %)
55	35	80	414
60	52	85	725
65	83	90	1256
70	138	95	2004
75	237	100	2978

Source: The IFID Centre calculations
Assuming 40m/60f (static) Annuity 2000 Table at 6% net interest.

60 Copyright © 2006 The IFID Centre

Table #2 displays these mortality credits at increasing ages assuming a unisex mortality table that averages the q values in equation (eq.1) with a 40% weight to male mortality and a 60% weight to female mortality. These numbers are generated based on the annuitant mortality table, which includes the anti-selection effect one would experience with such a group. Notice how it is only after age 65 that these credits exceed 100 basis points. Notice also how at higher ages they are quite substantial.

Recall that these mortality credits should also be viewed as a threshold investment return that is required to beat the income from the annuity during the year in question. If a so-called self-annuitizer can earn the pricing rate R, plus whatever is left of the mortality credits in equation (eq.1) after transaction costs, they are better off not annuitizing at age 50 and waiting until age 51 to reconsider the decision. The following equation (eq.2) makes this point algebraically.

$$a_{x+1} = a_x \left(\frac{1+R}{1-q_x} \right) - 1 \qquad \text{(eq.2)}$$

In this equation, the actuarial symbol a_x denotes the annuity factor, or the cost of $1 of income per year for life starting at age *x*, under a valuation rate of *R*. Likewise, a_{x+1} denotes the annuity factor at age *x+1*. Equation (eq.2) is an actuarial identity based on the definition of the annuity factor. From a financial point of view this implies the following. If the quantity a_x can be invested to earn a total return of $(1+R)/(1-q_x)$ or greater, the age-*x* lifecycle investor can consume the same dollar the annuity would have provided, and still have enough funds to purchase an identical income annuity, at age *x+1*, assuming the *q* and *R* values do not change from one year to the next. If that is the case, why would he or she ever annuitize at the age of x?

I am quite willing to concede that when the lifecycle investor has no bequest motive (i.e. does not care about leaving any inheritance) and experiences zero transaction costs, they should definitely annuitize at age 50 or even earlier, just to gain access to the albeit meager 42 basis points. But in the real world this never happens. Remember that while a 50 year old lifecycle investor might claim to have no bequest motives, these preferences might change over the remaining 30-40 years of life. Annuitizing at age 50 kills the option to develop bequest preferences. With a zero weight on the utility of bequest, there is only one way in which this can change.

Another problem with premature annuitization is that when the immediate annuity is of the fixed nominal (or even real) type – which currently represents 90% of income annuities sold in the U.S.[3] -- the lifecycle investor is selecting an *asset allocation* together with a *product allocation*. The asset class underlying the annuity is essentially fixed income bonds with a predetermined duration and sensitivity to interest rates. This is precisely where the irreversible nature of real-world annuities, as opposed to pure tontines, impacts the optimal age and process by which to annuitize. Given that this contract is for life, the annuitant must now commit to a fixed income asset allocation that can never be altered. A typical lifecycle investor would surely want to rebalance and reallocate their wealth to different asset classes over time, but the locked-in nature of

[3] According to LIMRA estimates, approximately $150 of retail premiums went to variable income annuities, while approximately $2 billion went to fixed income annuities, not including the 403(b) market.

the contract would impede the ability to rebalance. The inability to rebalance is costly, from a utility perspective and reduces the appeal of annuitization at all ages, as was argued in Browne, Milevsky and Salisbury (JRI, 2005). Stated differently and in the language of equation (eq.1), although we would like to gain access to the q_x portion, we might not desire the R that comes with it.

Indeed, in a perfect world one could offset, hedge or strip away the undesired exposure to bonds by shorting an appropriate fixed-income portfolio with an equal and opposite duration, but the transaction costs for the simple life-cycle investor would be prohibitive. Remember that by annuitizing prematurely into a fixed immediate annuity, not only is the investor forced into an undesired bond allocation today, they are forced to maintain this sub-optimal allocation for the rest of their natural lives. The increasing mortality credits might not be sufficient consolation for the sub-optimal asset mix. Imagine a life-cycle investor who rationally wants to maximize their discounted lifetime utility by holding 100% equities in their personal portfolio, or perhaps more given their human capital's hedging characteristics. Forcing them to hold bonds - even with the ongoing mortality credits - might be worse in a utility-adjusted sense, compared to holding their desired equities without the mortality credits.

In fact, even when a lifecycle investor has access to low cost variable immediate annuities that can be rebalanced amongst a variety of traditional asset class such as stocks, bonds, cash, real estate, commodities, etc., there is an inherent loss of flexibility that comes from restricting choices to a given company's family of investment accounts - something that is an inevitable byproduct of real-world annuities. All of these utility-destroying restrictions will negate the mortality credits at younger ages. This point has recently been made rigorously in the paper by Milevsky and Young (IME, 2006). That paper finds little theoretical justification for annuitization prior to the ages of 65/70.

Remember that financial economists and pension economists who argue the benefits of annuities and their mortality credit subsidies, as per equation (eq.1) are really discussing pure tontines which can be renegotiated at the end of some arbitrary

and fictitious time period. These products – if they actually existed – would contain the *real option* to change one's mind, preferences and strategy at the end of the period.

Along the same lines, I now address a related issue. How about annuitizing at a young age, but with income payments starting at an advanced age? Is this akin to annuitizing prematurely, or is it enough that income starts at an advanced age? Would the same deferral logic apply?

If the delayed annuity is 100% reversible -- i.e. the lifecycle investor can cash out at market value any time prior to the income commencement date -- then I would argue that annuitization has not really taken place. The purchase of the delayed annuity at age 30, 40 or even 50 is effectively a fixed income allocation with an embedded call option to annuitize. The call option's underlying state (stochastic) variable is the changing mortality tables used to price annuities or population hazard rate. Thus, since the lifecycle investor can fully reverse the decision, possibly subject to a market value adjustment based on the new level of interest rates, there is no pooling of mortality risk until the income annuity commences payment. The value of the embedded call option to annuitize – and hence the relative appeal of this type of product compared to a straight bond – would depend critically on the specifics, such as the implied mortality rates within the contract relative to current annuity rates. The devil is in the detail and it is impossible to pass judgment on the relative merits of such a product in the absence of the contract parameters[4]. Is the mortality table fixed at the time of purchase, or dependent on population mortality at the time of annuitization? Is the strike price of this option to annuitize currently in-the-money or far out-of-the-money? At best it could be a surrogate for a desired bond allocation, at worst it would be an overpriced and unnecessary call option.

In fact, I would argue that a traditional Defined Benefit (DB) pension plan is effectively a staggered purchase of fixed income strips, plus the mortality call options.

[4] See the working paper by Abaimova and Milevsky (IFID, 2005) for an attempt to analyze a type of product that offers this call option within Defined Contribution (DC) pension plans.

Note that even if the delayed annuity is not cashable in a mark-to-market sense, as long as there is some cash value that accrues over time and can be accessed at some point – perhaps at death, disability or retirement -- the same comments would apply. Oddly enough what we are saying is that a lifecycle investor within a DB pension plan is not really accruing or accumulating income annuities, but rather an option to annuitize based on some mortality table. The decision to annuitize takes place at retirement when the pensioner decides to not take the lump sum option, which is available in most DB pension plans.

Now, in contrast to the above, if the delayed annuity is 100% irreversible with zero cash value and no survivor benefit then I would concede that it might be optimal to start accumulating these credits at a young age, provided the above-mentioned concerns about transaction costs do not negate the mortality credits. This is the ALDA concept explored in Milevsky (NAAJ, 2005), where a small premium is paid on an ongoing basis in exchange for a mortality-contingent income that starts at an advanced age. Note, however, that the threshold for beating the implied return from the delayed annuity at younger ages would still be quite low, even if the income doesn't start until an advanced age. Another factor to consider would be the embedded option on a given mortality table – i.e. the commitment to use current rates regardless of what happens to aggregate mortality – which might also increase the relative value of such a strategy/product. Again, the devil is in the details and it is quite difficult to generalize about the merits of such products, especially once fees, commissions and profit margins are taken into account.

Thus, despite the preponderance of theoretical arguments in favor of annuitization, I am hesitant to advocate a single optimal age at which a life-cycle investor should convert his or her savings account into an irreversible income annuity. Given the many tradeoffs involved in this decision and numerous sources of uncertainty, I am quite comfortable arguing that annuitizing prior to age 60 is too early, while waiting until the age of 90 it is obviously too late. Indeed, at the advanced age of 90 the unisex

mortality rate of (q_{90}) 15% leads to mortality credits of 1850, which are insurmountable on any investment frontier.

This is why there is an emerging body of literature that is suggesting that lifecycle investors should annuitize slowly, akin to a dollar-cost averaging strategy[5]. Depending on contract and policy features this process would start at age 70 and continue until age 80 or 85 for example, until the entire amount of desired annuity income is actually annuitized. The paper by Milevsky and Young (IFID, 2005) proves the optimality of a staggered purchase option that annuitizes a small fraction on an ongoing basis. That result does not come from any attempt to speculate on interest rates or time the shape of the yield curve. Rather, it is the natural result from balancing out the competing risks we described above.

In sum, a number of previous research papers have been devoted to emphasizing the important role of income/payout annuities in the optimal portfolio of the lifecycle investor. I have repeated the arguments that product allocation to annuities (and insurance) is just as important as asset allocation to financial instruments such as stocks, bonds and cash. The theoretical arguments in favor of annuitization are so powerful that an entire body of economic literature has emerged under the title of "the annuity puzzle," which seeks to address the reasons why so few consumers actively and consciously embrace these instruments. Nevertheless, I remain agnostic on the precise age at which this product allocation should take place.

[5] DCA in general is a sub-optimal investment strategy, and has been shown to be mean variance inefficient by a number of writers.

References and Bibliography

Ameriks, J., R. Veres and M. J. Warshawsky (2001), “Making Retirement Income Last a Lifetime”, *Journal of Financial Planning*, December, Article 6 (www.journalfp.net)

Browne, S., M.A. Milevsky and T. Salisbury (2003) “Asset Allocation and the Liquidity Premium for Illiquid Annuities” , *Journal of Risk and Insurance*, Vol. 70(3), pg. 509-526.

Chen, P. and M.A. Milevsky (2003), “Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities”, *Journal of Financial Planning*, June, pg. 64-72

Davidoff, T., Brown, J. and Diamond, P. (2005), Annuities and individual welfare, *American Economic Review*, Vol. 75(2), pg.

Milevsky, M.A. (2005) “The Implied Longevity Yield: A Note on Developing an Index for Payout Annuities”, *Journal of Risk and Insurance*, 2005, Vol. 72(2), pg. 301-320

Milevsky, M.A. (2005) “Advanced Life Delayed Annuities: Pure Longevity Insurance with Deductibles”, *North American Actuarial Journal*, October, Vol. 9(4), pg. 109-122.

Milevsky, M.A. and A. Abaimova (2005) Variable Payout Annuities with Downside Protection: How to Replace the Lost Longevity Insurance in DC Plans, *IFID Centre Working Paper*, October 2005. (www.ifid.ca)

Milevsky, M.A. and V. Young (2004), Asset Allocation and Annuitization, *IFID Centre Working Paper*, www.ifid.ca

Yaari, M. (1965), "Uncertain Lifetime, Life Insurance and the Theory of the Consumer", *Review of Economic Studies*, Vol. 32, pg. 137-150.